First Order Perturbation Calculation for WQB Tracking Error

A thin quadrupole at location 1 with strength:

$q = B'L/B\rho = 1/f$ (H)

$-q$ (V)

- One WQB with tracking error 0.1% (10 units) at 120 GeV :

  $B'L = 19.6$ T/m $\times$ 2.134 m = 41.826 T-m/m

  $B\rho = 403.3945$ T-m

  $q = B'L/B\rho \times 0.1\% = 0.0001$ m^{-1}

  Lattice function at WQB (i.e., location 1):

  $\beta_1(x) = 57$ m

  $\beta_1(y) = 10$ m

  $D_1(x) = 0$ (in straight section)

- Tune change:

  $\Delta\nu(x) = q \times \beta_1(x) / 4\pi = 0.00045$

  $\Delta\nu(y) = -q \times \beta_1(y) / 4\pi = -0.00008$

- Beta-wave at location 2:

  $\Delta\beta_2(x)/\beta_2(x) = -q \times \beta_1(x) \times \sin 2\psi(x) \leq -0.0057$

  $\Delta\beta_2(y)/\beta_2(y) = q \times \beta_1(y) \times \sin 2\psi(y) \leq 0.001$

  $\psi = \mu_2 - \mu_1$ = phase advance from 1 to 2

- Dispersion wave at location 2:

  $\Delta D_2 = -q \times D_1 \times \{\beta_1(x) \times \beta_2(x)\}^{1/2} \times \sin\psi(x) = 0$

  (Note that the beta-wave frequency is twice that of the dispersion wave.)

- Seven WQB's:

  $\Delta\nu(x) = 7 \times 0.00045 = 0.00315$

  $\Delta\nu(y) = 7 \times -0.00008 = -0.00056$

  $\Delta\beta_2(x)/\beta_2(x) = \sum [-0.0057 \times \sin 2\psi(x)]$

  $\Delta\beta_2(y)/\beta_2(y) = \sum [0.001 \times \sin 2\psi(y)]$